NKS SUMMER SCHOOL 07

Minimal Models of the Complexity of Financial Security Prices

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Introduction

What's the simplest rule to model the complexity and randomness of financial markets? Security prices have complex behavior in the time series and cross section of prices, returns, volume, and liquidity. It's easy to mimic security prices by introducing randomness, either externally like a random walk or internally through large structures such as cellular automata involving trade between many investors, but what are the *minimal* models that generate complexity?

One Trader

I find that a single investor trading in a single security can generate complex price behavior all by himself. He makes trading decisions by looking back at the signs of the price change over the past few days. Because he is the representative investor, the price adjusts with his decisions.

The Simplest Rule with Complex Behavior

Rule #54 is the only interesting 2-state buy/sell rule with lookbacks up to 15. With fixed lookbacks, the price series will always cycle, but rule 54 often achieves near maximal cycle lengths, which can be quite long. For example, a 15-business-day lookback will cycle in about 130 years.

More Complicated Rules

Surprisingly, allowing an infinite lookback window, so that the trader looks back over each price change in the security's history back to its very first day, *removes* complexity. There are *no* rules with either two or three internal states that produce interesting behavior: all cycle very quickly. Allowing the lookback window to grow on a log scale, so that the trader looks back one more day before he is likely to cycle, does produce complex behavior arbitrarily far, though sometimes with long periods of high predictability interspersed.

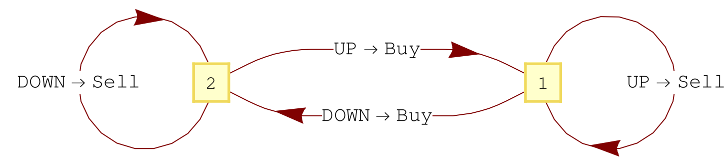
Multiple Traders

When several rules trade in the same security, we can also generate the trading volume and measures of liquidity. Sometimes complex rules trade and remove complexity and sometimes simple rules trade and create complexity. Liquidity falls, even though volume is constant, before a crash.

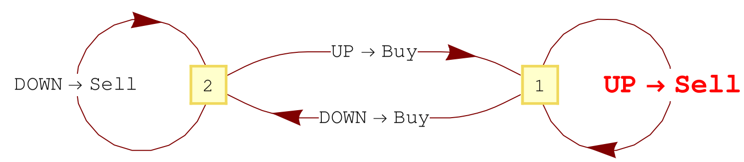
Our trader comes into work on Thursday and sees the following history for the market.

Monday: Down **Tuesday: Up** **Wednesday: Up**

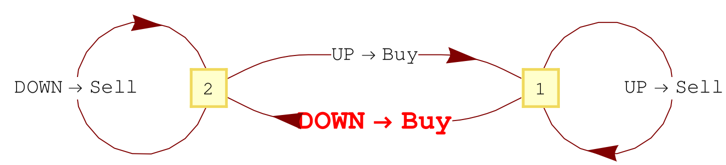
Our trader is modeled by the following rule. He starts in state 1 at the beginning of each new day.



Wednesday was UP and he starts in state 1, so he follows the UP arrow back to state 1:



Tuesday does the same thing. But Monday was down, so he follows the DOWN arrow from state 1 :

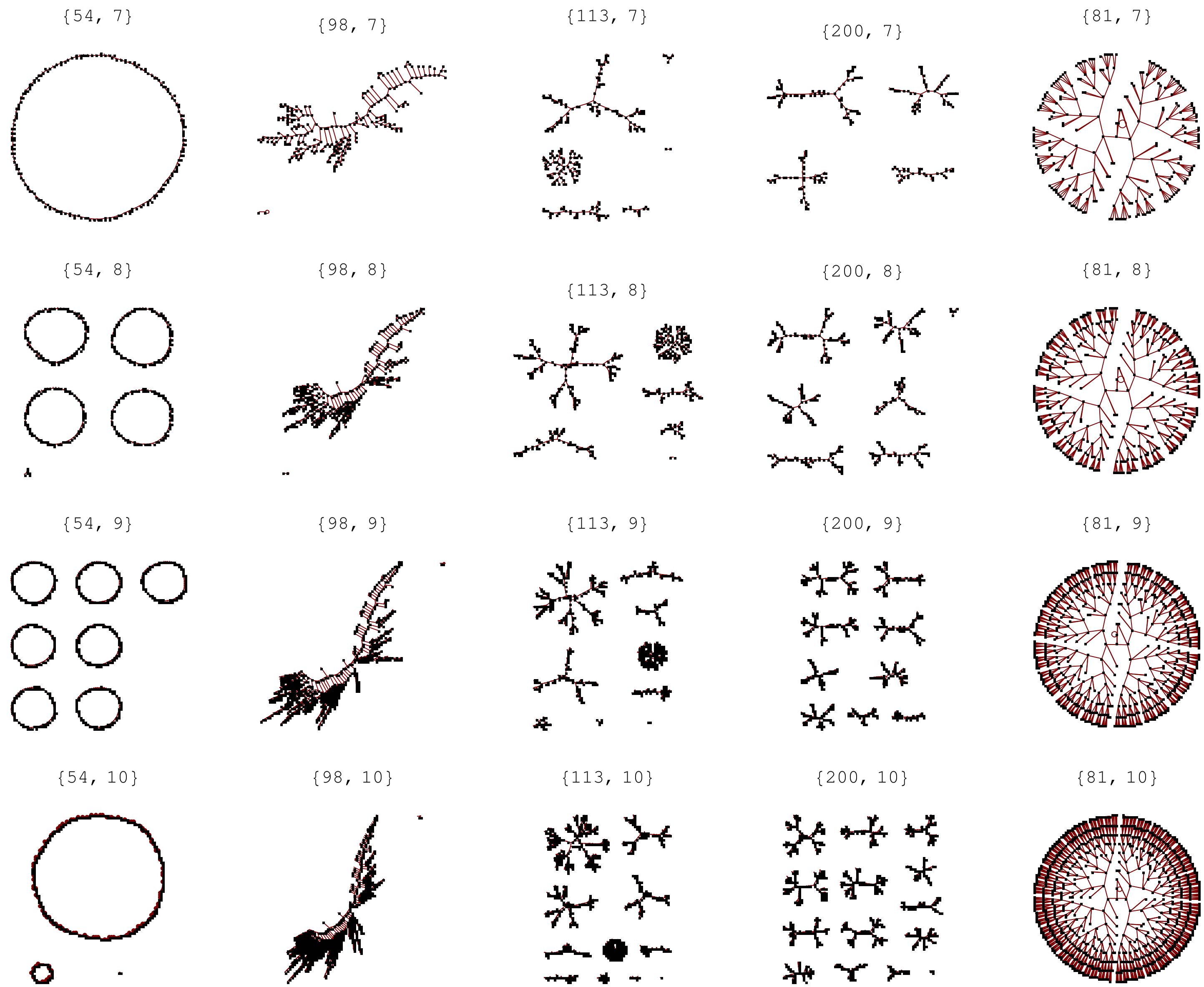


This is the last price change he looks at so his output on the arrow, in this case **Buy**, is his order for the market. The market goes up, and when he comes in on Friday, here is what he sees::

Tuesday: Up **Wednesday : Up** **Thursday: Up**

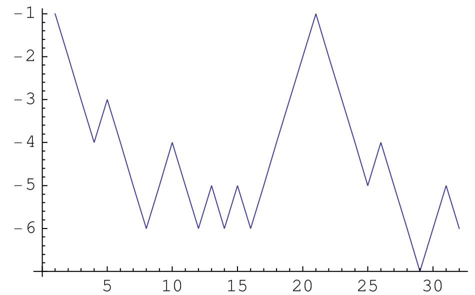
He follows the arrows again. (This time, he sells.)

In these representative transition diagrams for 2-state, buy/sell rules, each point represents a possible sequence of ups and downs of length 7, 8, 9, or 10.

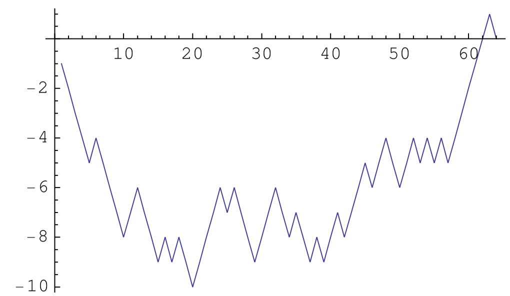


Rule 54 often has close to maximal cycle length. Here are the first $\mathbf{2^n}$ time steps for lookback windows of $\boldsymbol{n} = \mathbf{5, 6, 7, 10, 11, 12, 13, 14, 15}$. (For $\boldsymbol{n} = \mathbf{7}$ **or** $\mathbf{8}$, rule 54 is simple.)
Captions show {**rule, {# states, # actions}, {period, maxperiod}**}. The closer the period is to the maximum, the closer the ending point of the price series will be to the starting point.

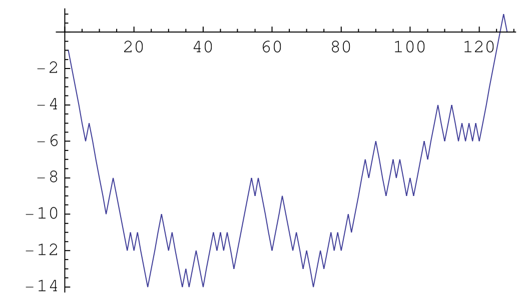
{54, {2, 2}, {21, 32}}



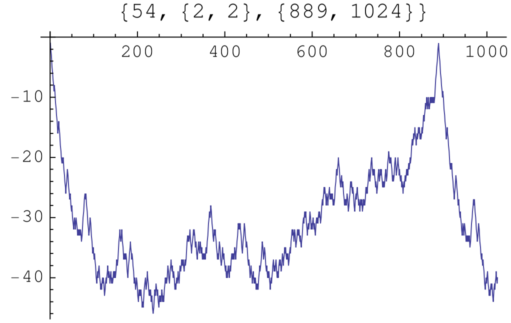
{54, {2, 2}, {63, 64}}



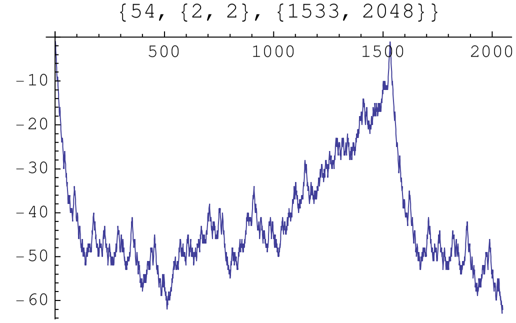
{54, {2, 2}, {127, 128}}



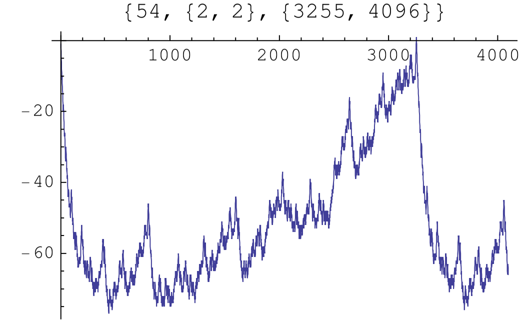
{54, {2, 2}, {889, 1024}}



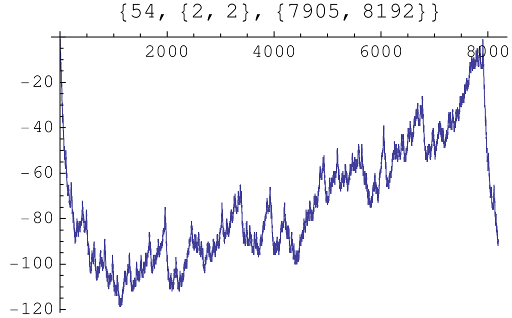
{54, {2, 2}, {1533, 2048}}



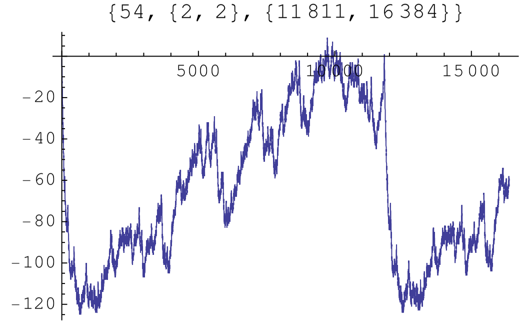
{54, {2, 2}, {3255, 4096}}



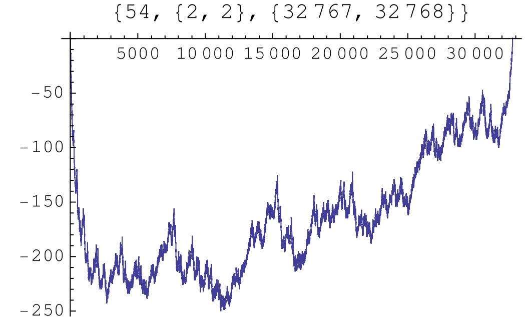
{54, {2, 2}, {7905, 8192}}



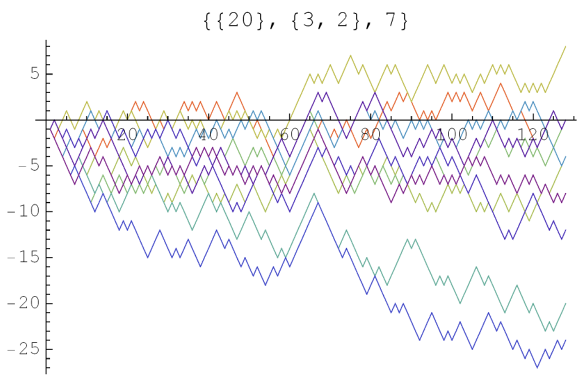
{54, {2, 2}, {11 811, 16 384}}

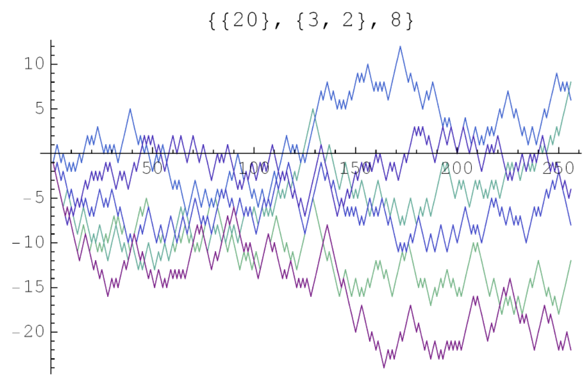


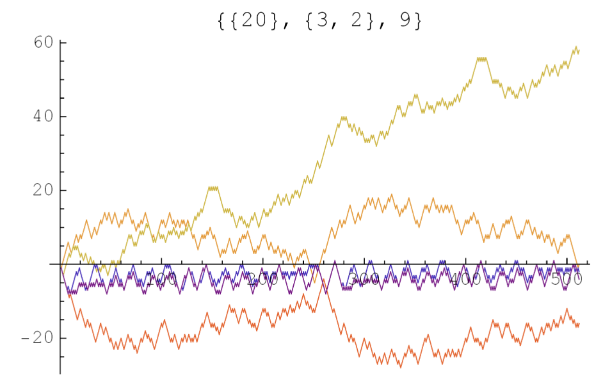
{54, {2, 2}, {32 767, 32 768}}



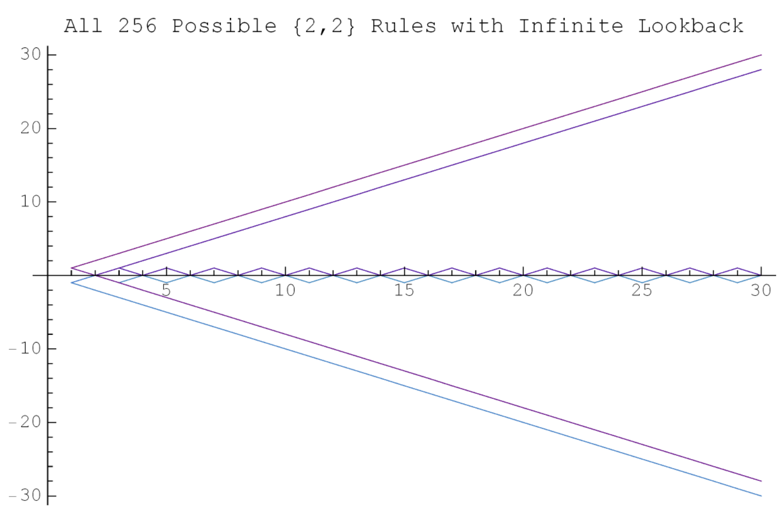
Among 3 state, buy/sell rules, variety emerges. Here are about 20 of the most interesting 3-state, buy/sell rules with lookbacks of 7, 8, or 9 days.

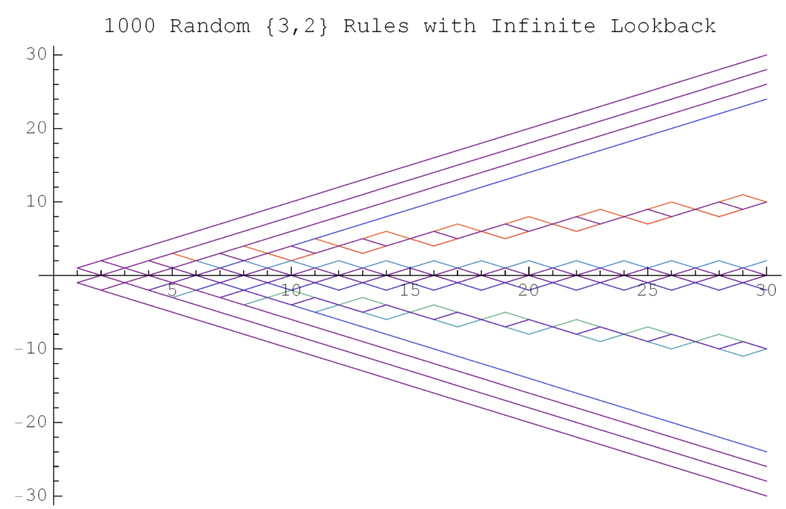






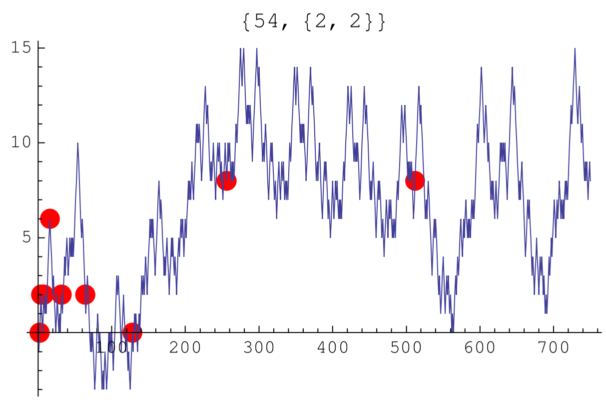
Does infinite lookback add complexity? Unfortunately, *every* 2- or 3-state rule with infinite lookback generates only repetitive behavior.

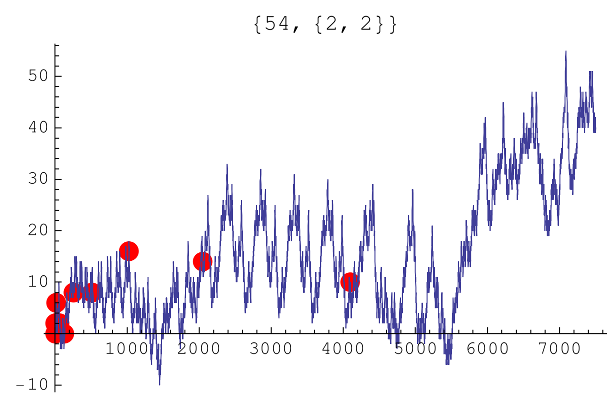


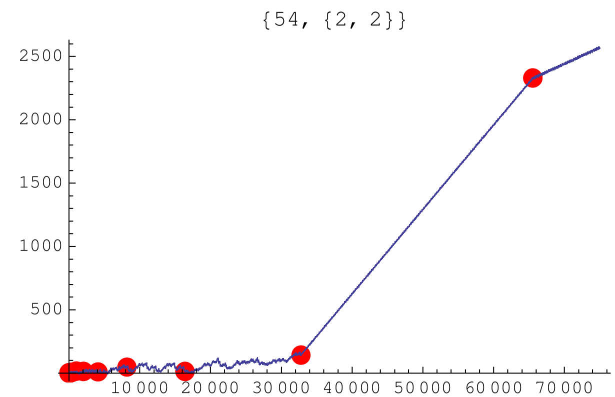


Another type of growing lookback increases the number of days the trader considers based on the length of the time series so far. So when there have been $\mathbf{2^n}$ time steps in the past, he now has a lookback of $\boldsymbol{n}$.

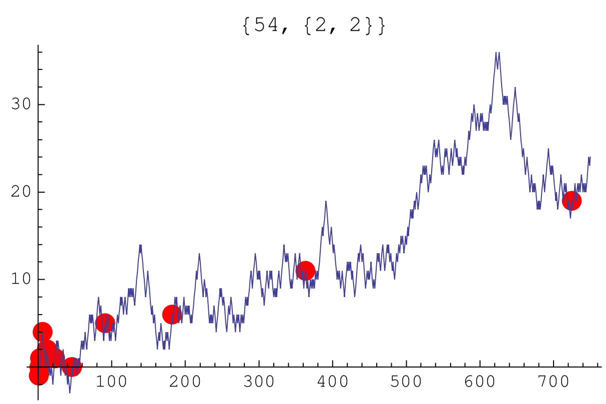
Rule 54 gets "stuck" in a short cycle after 32 768 steps and doesn't come out of it even after looking back more at 65 536 steps. The red dots represent times when the trader increments his lookback window. The three graphs differ only in how many time steps they show (750, 7500, and 75 000).

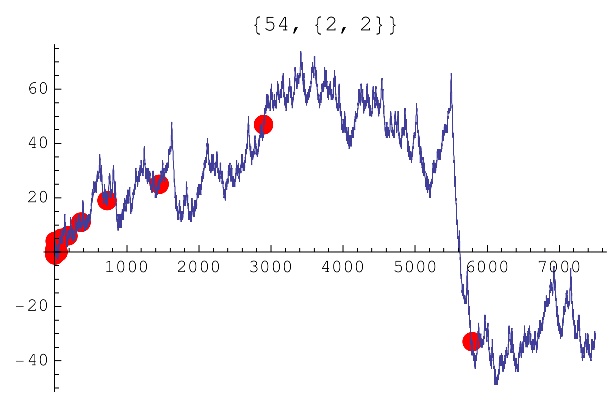


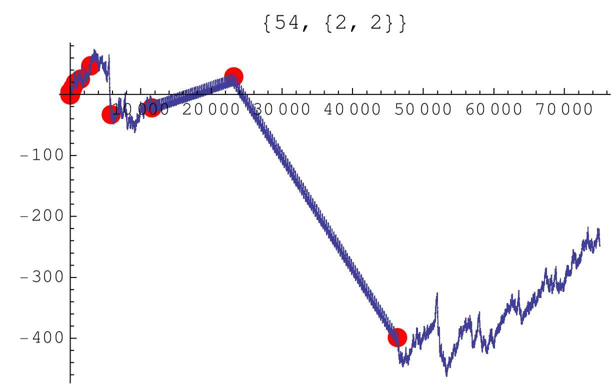




We can increment the lookback window so it cycles, for example, halfway through the period length instead of at the end, or one period in advance. Here is a graph when it cycles 1.5 periods earlier than it otherwise would. Note the periods of predictability surrounded by complexity.

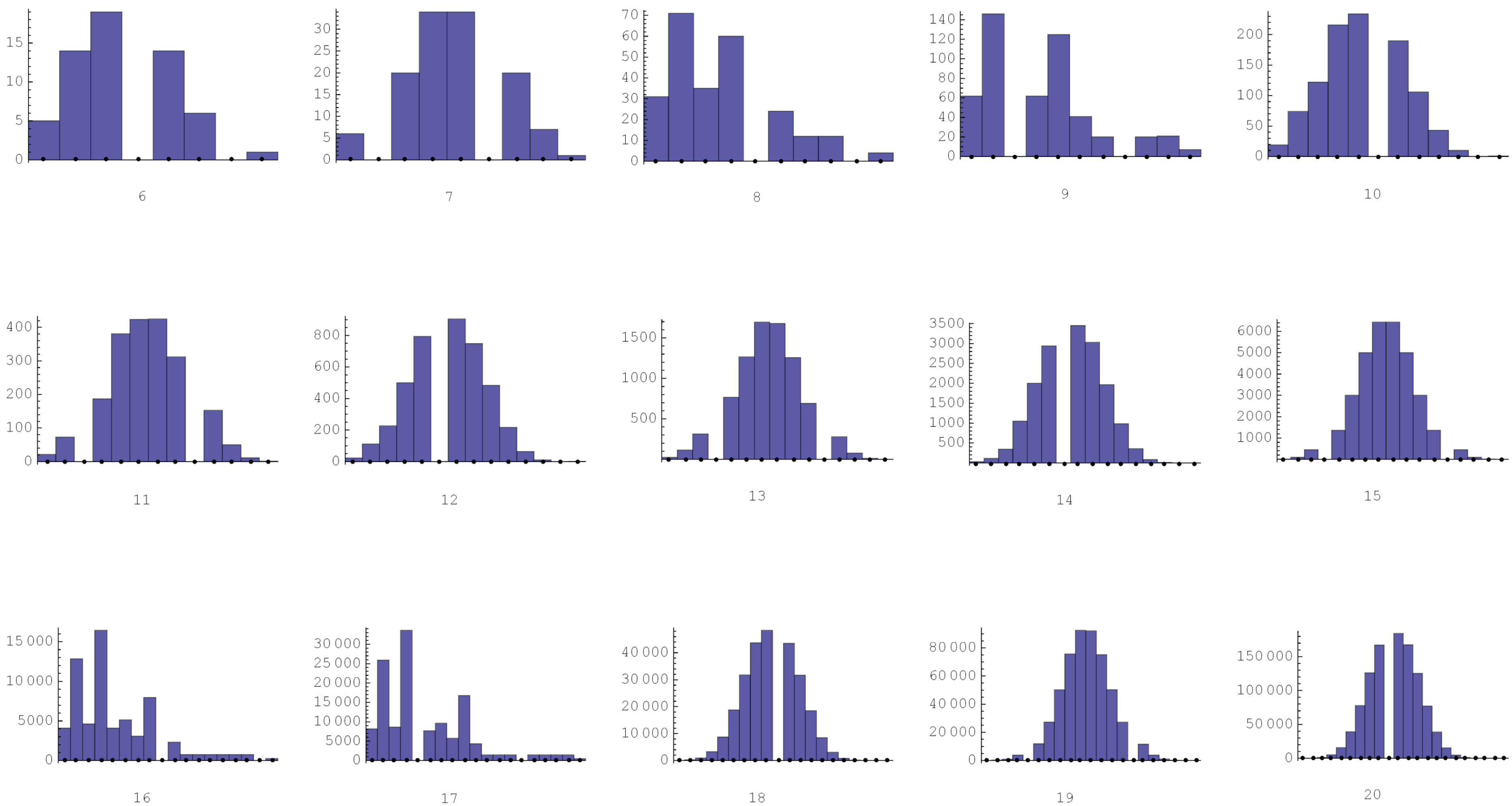






The statistical properties of the time series generated by these simple rules are not the same as random walks.

Here are plots of the histograms of the moving averages of the daily changes of rule 54.



Top: When complex rules trade, sometimes simpler behavior emerges. Bottom: When simple rules trade, sometimes more complex behavior emerges. Also note liquidity can drop while volume remains constant.